Trading frequency and the efficiency of price discovery in a non-dealer market

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The increasing popularity of non-dealer security markets that offer automated, computer-based, continuous trading reflects a presumption that institutionally-set trading sessions are economically obsolete. This theoretical paper investigates the effect of the trading frequency, a key feature of the trading mechanism, on the efficiency of price discovery in a non-dealer market. By tracing the market pricing error to the correlation structures of arriving information and pricing errors of individual traders, the effect of diverging expectations on error-based and overall return volatility is isolated. The analysis reveals that, due to a portfolio effect, an increase in the trading time interval has contradictory effects on the portion of return volatility stemming from pricing errors. The greater accumulation of information increases error-based return volatility, but the greater volume and number of traders per session have the opposite effect. The net effect on overall return volatility can go either way. It is found that the return volatility of heavily traded securities is likely to be minimized under continuous trading, but that of thinly traded securities may be minimized under discrete trading at moderate time intervals. The latter is more likely to occur the greater is the divergence of expectations among traders. These findings challenge the presumption that automated continuous trading in a non-dealer market is more efficient than discrete trading for all securities, regardless of trading volume. The findings are applicable to all economies, but have special importance for developing countries where typically a single market is dominated by small issues and a low volume of trade. As a by-product of the analysis, it is shown how to correct the biased estimate of inter-session price volatility when observations are less frequent than the trading sessions themselves.

Keywords: trading frequency, non-dealer security markets, price discovery, portfolio effect, return volatility

1. INTRODUCTION

The extensive research of recent years teaches us that the choice of a trading mechanism has important consequences for price formation and market liquidity. Although nearly all published studies assume the presence of market
makers and specialists, a significant number of exchanges today operate without market makers. Many of the non-dealer markets, as the latter are referred to, are now fully automated and continuously run; they include security exchanges in developed and emerging economies. Without the benefit of experience or a relevant body of theory, these markets are faced with the task of accommodating, side by side, high-volume securities held and traded by many and low volume securities traded by few. At one end of the spectrum, the majority of markets offer continuous trading that is open to all listed securities regardless of trading volume; at the other end, few markets like the Brussels Stock Exchange offer separate trading sessions of different length and frequency, and possibly a different trading mechanism, to groups of securities segregated by trading volume.

This paper is written in search of a theory relating a security's pricing efficiency to its trading volume and institutionally-set trading frequency in a non-dealer market. We begin our analysis with the theoretical framework of earlier writers who do not explore these issues. Garbade and Silber (1979a, 1979b), Amihud and Mendelson (1987), and Hasbrouck (1993), describe the equilibrium trading price at any time as a sum of two components: the intrinsic value and the average pricing error of individual traders. To explore the effects of the trading frequency on pricing efficiency, we generalize their models by relaxing their simplifying extreme assumptions regarding the correlation coefficients among the pricing errors of individual traders, and between those errors and arriving new information. We treat those correlations as market/security-unique parameters and examine their effects on the relationship between trading frequency and return volatility, and on the speed of price adjustment to new information.

Our new focus on the structure of trader pricing errors reveals that an increase in the trading time interval has contradictory effects on the portion of return volatility caused by pricing errors. On the one hand, the greater accumulation of new information over an extended time interval increases error-based return volatility. On the other hand, the increased volume and number of traders per session decrease the aggregate pricing error and related return volatility due to a portfolio effect. The net effect on the overall return volatility can go either way. We explore the conditions determining the optimal trading interval under which the overall return volatility is minimized. Consistent with conventional wisdom, we find that the volatility of heavily traded, widely held securities is likely to be minimized under continuous trading; in contrast, the volatility of thinly traded, narrowly held securities may be minimized under continuous or discrete trading at moderate time intervals. The latter is more likely to occur the greater is the divergence of expectations among traders. As a by-product of the analysis, we demonstrate the existence of a bias in estimating inter-session return volatility from observations that are less

frequent than the trading sessions themselves. We go on to propose a method for removing that bias.

The remainder of this paper is organized as follows. Section 2 contains the theoretical model and its implications for the selection of an optimal trading frequency. Section 3 analyses the relationship of our model to earlier research. Section 4 discusses implications of our results for the measurement of inter-session price volatility. Section 5 offers a summary and conclusions.

2. THE MODEL

Following Garbade and Silber (1979b), consider a market for a given security in which $K$ traders transact at the end of each time period $t$ ($t = 0, 1, \ldots$) assumed for convenience to be of a unitary length. Let each trader $k$ ($k = 1, 2, \ldots, K$) have an initial endowment $E$ of the security, and a gross periodic demand at $t$ of

$$D_{kt} = E + a(P_{kt} - P_t)$$

where $P_{kt}$ is the trader's reservation price, $P_t$ the observed market price, and $a$ the slope of the demand schedule. The aggregate demand flow is

$$\sum_{k=1}^{K} D_{kt}(P_t) = KE + a \sum_{k=1}^{K} P_{kt} - KaP_t$$

and market clearing implies equality of aggregate demand and supply

$$KE = KE + a \sum_{k=1}^{K} P_{kt} - KaP_t$$

and this solution for the equilibrium price

$$P_t = \frac{\sum_{k=1}^{K} P_{kt}}{K}$$

(4)

The individual trader's reservation price is expressed as a sum of the security's intrinsic value at $t$, $m_t$, and the trader's pricing error, $\tilde{h}_{kt}$

$$P_{kt} = m_t + \tilde{h}_{kt}$$

(5)

where the intrinsic value randomly changes with the arrival of new information signals according to

$$m_t = m_{t-1} + \tilde{w}_t$$

(6)

subject to the condition that both $\tilde{h}_{kt}$ and $\tilde{w}_t$ are serially uncorrelated, each with a zero expected value. Substitution of (5) in to (4) yields

$$P_t = m_t + \frac{\sum_{k=1}^{K} h_{kt}}{K}$$

(7)
where the second term is the deviation of the market transaction price from the intrinsic value.

Other things equal, if the pricing errors of individual traders, \( h_{kt} \), are less than perfectly correlated, an exogenous increase in the number of traders per session, \( K \), would bring the equilibrium price closer to the intrinsic value. Given the constant periodic flow of traders seeking to transact, the number of transacting traders per session is proportional to the time interval set between sessions. An increase in that interval from its initial unitary length to \( T > 1 \) would increase the number of traders per session from \( K \) to \( KT \). Similar to the derivation of (7), the new equilibrium price at \( t \) is

\[
P_t = m_t + \frac{\sum_{k=1}^{TK} h_{kt}}{TK}
\]

(8)

where the aggregate pricing error is smaller than in (7) due to error diversification. Recalling (6), the intrinsic value at \( t + T \) is

\[
m_{t+T} = m_t + \sum_{s=t+1}^{t+T} \tilde{w}_s
\]

(9)

Based on (8) and (9), the equilibrium price at \( t + T \) is

\[
P_{t+T} = m_t + \sum_{s=t+1}^{t+T} \tilde{w}_s + \frac{\sum_{k=1}^{TK} h_{k,t+R}}{TK}
\]

(10)

The difference between the intrinsic value and the transaction price in (8) or (10), which measures the error in market pricing at a point in time, is an index of the market’s inefficiency in price discovery. To express the concept of inefficiency in terms of return volatility, we first state the price change over \( T \) by subtracting (8) from (10)

\[
\Delta P_T = P_{t+T} - P_t = \sum_{s=t+1}^{t+T} \tilde{w}_s + \frac{\sum_{k=1}^{TK} h_{k,t+T}}{TK} - \frac{\sum_{k=1}^{TK} h_{kt}}{TK}
\]

(11)

Suppose an investor who observes \( P_t \) at time \( t \) makes at that time a decision to buy or sell at time \( t + T \). As viewed by this investor, pricing inefficiency is expressed in (11) by the portion of the price change traced to market ignorance, measured by the algebraic sum of the last two terms. To restate (11) in return volatility parameters, we introduce two assumptions which replace the more restrictive ones used by our predecessors. The first assumption describes the cross-sectional relationship among pricing errors of individual traders...
Assumption 1: \( \text{cov}(\tilde{h}_{k,t}, \tilde{h}_{m,s}) = \begin{cases} \rho \sigma_h^2 & \text{if } t = s \\ 0 & \text{if } t \neq s \end{cases} \)

where parameter \( \sigma_h^2 = \text{var}(\tilde{h}) \) measures the dispersion of pricing errors among traders, and \( \rho \) the correlation between pricing errors of individual traders. The second assumption describes the relationship between pricing errors and arriving information:

Assumption 2: \( \frac{\text{cov}(\tilde{h}_{k,t+T}, \tilde{w}_s)}{\sigma_h \sigma_w} = \begin{cases} \rho_{hw} & \text{if } t < s \leq T \\ 0 & \text{otherwise} \end{cases} \)

where parameter \( \sigma_w^2 = \text{var}(\tilde{w}) \) measures the volatility of arriving information, and \( \rho_{hw} \) the correlation between arriving information and pricing errors of individual traders. Our treatment extends the works of Garbade and Silber (1979a, 1979b), Amihud and Mendelson (1987), and Hasbrouck (1993) by recognizing that parameters \( \rho \) and \( \rho_{hw} \) may be different from zero (see Section 3 below). Under our less restrictive assumptions, the variance of the price difference in (11) is

\[
\text{Var}(\Delta P_T) = T \sigma_w^2 + 2 \frac{\sigma_h^2}{TK} (1 - \rho) + 2 \rho \sigma_h^2 + 2T \rho_{hw} \sigma_h \sigma_w
\]

where the contribution of pricing errors to the overall price variance over \( T \) is summed by the last three terms. The positive effect of the trading interval, \( T \), on the error-based and overall volatility comes from the first and last terms; the negative effect comes from the second term where \( T \) is in the denominator.

Without a general equilibrium model, we have no welfare function to guide us in selecting policy objectives. The most obvious and simplest options are the minimization of overall volatility or error-based volatility by including or excluding the first term. Following Hasbrouck (1993), we adopt the partial-equilibrium policy objective of maximizing liquidity by minimizing the overall volatility of a security’s return between trading sessions. Since of the two objectives the minimum error-based volatility occurs at a longer or equal trading interval, our chosen objective works in favour of continuous trading and against our own claim that discrete trading may be optimal.

According to (12), an increase in the time interval, \( T \), between trades has contradictory effects on the return variance between sessions: the variance increases by the greater accumulation of new information (first and last terms), and decreases by the diversification of pricing errors among more traders per session (second term). The extent of the second effect depends on the correlation between the pricing errors of individual traders. With full unanimity among traders, the extreme value \( \rho = 1 \) implies no opportunity for decreasing error-based return volatility. In contrast, the value \( \rho < 1 \) may create the opportunity for decreasing the volatility from this source, and possibly the overall return volatility. To find the trading interval that minimizes the overall...
return volatility, the derivative of (12) with respect to \( T \) is set at zero and solved for \( T = T^* \)

\[
T^* = \frac{\sqrt{2} \sigma_h \sqrt{1 - \rho}}{\sqrt{K} \left( \sigma_w^2 + 2 \rho_{hw} \sigma_h \sigma_w \right)} = \sqrt{\frac{2(1 - \rho)}{K(g^2 + 2 \rho_{hw} g)}}
\]

where \( g = \frac{\sigma_w}{\sigma_h} \). Equation 13 shows that for \( \rho < 1 \), the minimum-variance trading interval monotonically decreases with an increase in \( K \), the periodic flow of traders. Since under a given trading frequency, \( K \) is a proxy for a security's trading volume, continuous trading is more likely to minimize the return volatility of widely held, actively traded securities when characterized by high values of \( \rho \), \( \rho_{hw} \), and the ratio \( \sigma_w/\sigma_h \). Symmetrically, continuous trading is less likely to minimize the return volatility of thinly traded, small issues that lack wide distribution, especially when characterized by low values of the same parameters. To gauge the importance of these relations, we turn to numerical examples.

Figure 1 displays the effect of the trading interval on the overall return volatility stated by (12) and the minimum-volatility trading interval stated by (13) under alternative values of parameters \( g, \rho \) and \( \rho_{hw} \). Panel 1(a) shows that \( \rho_{hw} \) has a negligible effect on volatility and the optimal trading interval. Panels 1(b)–1(d) are designed to reveal the roles of \( g \) and \( \rho \) in this relationship. A decrease in \( g = \sigma_w/\sigma_h \) across panels is seen to cause an increase in the optimal trading interval. At one extreme, as \( \rho \) approaches 1, continuous trading minimizes volatility regardless of the trading volume; at the other extreme, as \( \rho \) approaches 0, volatility is minimized at a longer trading interval. The special case of a monotonically decreasing volatility, as under \( \rho = 0 \) in panel 1(d), describes a scenario of a security that would not be listed on the exchange.

3. USING THE RESULTS TO REINTERPRET EARLIER MODELS

Hasbrouck (1993) assumes the following relationship between the individual trader's pricing error \( \tilde{h}_{k,t+T} \) and the news \( \tilde{w}_s \)

\[
\tilde{h}_{k,t+T} = \alpha \sum_{s=t+1}^{t+T} \tilde{w}_s + \varepsilon_{k,t+T}
\]

where \( \varepsilon_{k,t+T} \) is white noise, \( \sigma^2_\varepsilon = \text{Var}(\varepsilon_{k,t+T}) \), and the coefficient \( \alpha \) decomposes the pricing errors into information-correlated errors (\( \alpha > 0, \varepsilon = 0 \)) and information-uncorrelated errors (\( \alpha = 0 \)). Using this notation, our market pricing error at time \( t + T \) is obtained again by averaging individual traders' errors over TK traders.

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2 In the special case \( \rho = 0 \) and \( \rho_{hw} = 0 \), this result is similar to Garbade and Silber's (1979b), leading to a corner solution. The return variance in (12) is minimized under continuous trading (\( T^* = 0 \)) or the security is de-listed (\( T^* = \infty \)).
This equation, which corresponds to Hasbrouck's expression (1993, Equation 3) for the relationship between pricing errors and changes of intrinsic value, is used below to reinterpret earlier models in light of our results.

\[
\frac{1}{TK} \sum_{k=1}^{TK} h_{k,t+T} = \alpha \sum_{k=1}^{TK} e_{k,t+T} + \frac{\alpha}{TK} \sum_{k=1}^{TK} \tilde{w}_t
\]  

(15)

Fig. 1. Trading interval and return volatility

Return volatility is simulated by

\[
\text{Var}(\Delta P_t) = T\sigma_w^2 + 2 \frac{\sigma_n^2}{T} (1 - \rho) + 2\rho\sigma_n^2 + 2T\rho_{nw}\sigma_n\sigma_w
\]

where \( T \) is the length of trading interval in minutes. In all panels \( K = 100, \sigma_n = 0.001 \) and \( g = \sigma_w / \sigma_n \). In panel 1(a), \( \rho = 0.5, \rho_{nw} = 0.0, 0.5, 1.0, \) and \( g = 30 \). In panel 1(b), \( \rho = 0.0, 0.5, 1.0, \rho_{nw} = 0.5, \) and \( g = 30 \). In panel 1(c), \( \rho = 0.0, 0.5, 1.0, \rho_{nw} = 0.5 \) and \( g = 15 \). In panel 1(d), \( \rho = 0.0, 0.5, 1.0, \rho_{nw} = 0.5, \) and \( g = 5 \).
In a case considered by Hasbrouck (1993) following Beveridge and Nelson (1981), the pricing error is fully correlated with the change of intrinsic value. The scenario combining the conditions $\alpha \neq 0$ and $\sum \varepsilon_{k,t} / TK = 0$ is reproduced by (15) when the effect of the errors $\varepsilon_{k,t+\tau}$ is eliminated through diversification over a large number of traders.

In another extreme case considered by Hasbrouck (1993) following Watson (1986), pricing errors are uncorrelated with new information, a scenario reproduces by substituting $\alpha = 0$ in (15). Given (14), $\Delta P_T$ in (12) is restated as

$$\Delta P_T = \sum_{s=t+1}^{t+T} \hat{w}_s + \sum_{k=1}^{TK} \alpha \sum_{s=t+1}^{t+T} \hat{w}_s + \varepsilon_{k,t+T} - \sum_{s=t-T+1}^{t+T} \alpha \hat{w}_s - \sum_{k=1}^{TK} \varepsilon_{k,t}$$

(16)

and the variance of price changes as

$$\text{Var}(\Delta P_T) = [1 + 2(\alpha + \alpha^2)] T \sigma_w^2 + 2 \sigma_e^2 / TK$$

(17)

This formulation differs from Hasbrouck’s (1993) since our error variance, $\sigma_e^2$, is attributed to individual traders. The case of pricing errors that are uncorrelated with new information corresponds to the assumptions of Garbade and Silber (1979a). It implies that both $\rho_{hu}$ and $\rho$ are a function of $\alpha$ and $T$. Specifically, (17) can also be obtained by substituting $\rho = (\alpha^2 T \sigma_w^2) / (\alpha^2 T \sigma_w^2 + \sigma_e^2)$ and $\rho_{hu} = \alpha \sigma_w^2 / (\alpha^2 T \sigma_w^2 + \sigma_e^2)$ in (12). As $T \to \infty$, $\rho$ increases approaching unity and $\rho_{hu}$ decreases approaching zero. Furthermore, when $\alpha > 0$ (or $\rho_{hu} > 0$) traders’ reaction amplifies the effect of the news.

Note further that under Hasbrouck’s model, the scenario combining $\alpha \neq 0$ and $\sum \varepsilon_{k,t} / TK = 0$ is joined with the assumptions $\rho = 1$ and $\rho_{hu} = 1 / T$ specifying unanimity among traders and correlation (inversely dependent on the trading frequency) between traders’ pricing errors and arriving information. In contrast, the assumption $\alpha = 0$ made by Garbade and Silber (1979b), Amihud and Mendelson (1987), and Damodaran (1993) implies that pricing errors are uncorrelated among individual traders ($\rho = 0$) or with arriving information ($\rho_{hu} = 0$). Finally, using Hasbrouck’s notation, we derive the optimal trading interval by setting to zero the derivative of (17) with respect to $T$ and solving for $T = T^*$

$$T^* = \frac{\sigma_e}{\sigma_w \sqrt{K} [1 + 2(\alpha + \alpha^2)]}$$

The same holds for Garbade and Silber (1979b).
4. IMPLICATIONS FOR VOLATILITY MEASUREMENT

Since the time interval \((t, t + T)\) in (17) is the interval between trading sessions, the measurement of return volatility should preferably be performed on data of consecutive transactions. In practice, however, empirical studies often rely on less frequent observations such as market closing prices. In the analysis below, we show why return volatility estimates based on less frequent observations are biased, and how the bias can be removed.

Let \(\hat{T}\) stand for the observation time interval where \(T \leq \hat{T}\) is the trading interval. We demonstrate next that the estimated variance caused by the error term is independent of \(\hat{T}\), a result stemming from Assumption 2 (Equation 14). According to this assumption, the pricing error of each investor is correlated only with news which affects the security price for the first time. The price difference over the measurement interval \((t, t + \hat{T})\) is given by

\[
\Delta P_{\hat{T}} = P_{t+\hat{T}} - P_{t} = \sum_{s=t+1}^{t+\hat{T}} \tilde{w}_s + \sum_{k=1}^{N} \alpha \sum_{s=t+\hat{T}-T+1}^{t+\hat{T}} \tilde{w}_s + \varepsilon_{k,t+\hat{T}} - \sum_{s=t-T+1}^{t} \alpha \sum_{s=t}^{t} \tilde{w}_s - \varepsilon_{k,t}\n
\]

and the return variance by

\[
\text{Var}(\Delta P_{\hat{T}}) = \hat{T} \sigma_w^2 + 2(\alpha + \alpha^2)T \sigma_w^2 + \frac{2 \sigma_e^2}{TK}\n
\]

The first term in (19) is the variance of the change in the intrinsic value, here a function of the measurement interval, \(\hat{T}\). The only difference between (19) and (17) is in the time interval used in the first term: unlike (19), the interval used in (17) corresponds to the actual trading. The second and third terms in (19), which contain the pricing errors and their covariance with changes in the intrinsic value, are independent of \(\hat{T}\). Based on a comparison between the two equations, the sought return variance corresponding to the trading interval, \(T\), can be estimated from observations conveniently collected for a longer time interval, \(\hat{T}\), by

\[
\text{Var}(\Delta P_T) = \text{Var}(\Delta P_{\hat{T}}) - \sigma_w^2(\hat{T} - T)\n
\]
pricing error of individual traders. We show that due to the diversification of trader errors – a factor omitted by previous writers – the market pricing error is inversely related to the number of traders. An increase in the time interval between trading sessions increases the number of traders per session, thereby decreasing the average pricing error. The increased time interval between trading sessions has conflicting effects on return volatility: the ameliorating effect of error diversification works against the effect of greater accumulation of new information due to less frequent trading. Our findings suggest that continuous trading is more likely to minimize the overall return volatility of securities traded at a high volume; but the return volatility of thinly traded securities may be minimized under continuous trading or discrete trading at moderate time intervals. The latter is more likely to occur when there is a low correlation among the pricing errors of individual traders. These findings challenge the common presumption that continuous trading is superior for all securities, regardless of trading volume. They suggest the need for empirical studies to compare the efficiency of continuous and discrete trading systems for thinly traded securities.

A by-product of the analysis is a formula for correcting the bias inherent in estimating the return volatility between trading sessions from less frequent observations. That formula does not depend on the correlation of pricing errors among individual traders.

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